GROUP IN LOGIC AND THE METHODOLOGY OF SCIENCE PRELIMINARY EXAMINATION

There are eight questions. Partial credit may be assigned for substantially correct partially worked solutions. To pass, you need a score of roughly fifty percent, though the ultimate decision about the passing mark will be decided by the committee of graders.

1. Prove or refute:

(a) If A and B are disjoint Σ^0_1 subsets of ω, then there is a Δ^0_1 set C such that $A \subseteq C$ and C is disjoint from B.

(b) If A and B are disjoint Π^0_2 subsets of ω, then there is a Δ^0_2 set C such that $A \subseteq C$ and C is disjoint from B.

2. Let T be a decidable theory in a finite language, and suppose all models of T are infinite. Show T has a model $\mathfrak{A}$ with universe ω such that the satisfaction relation $\{(\phi, \overrightarrow{n}) | \mathfrak{A} \models \phi[\overrightarrow{n}]\}$ is recursive.

3. Show that there is a model $\mathfrak{N} \models \mathrm{PA}$ of Peano arithmetic and $a \in |\mathfrak{N}| \smallsetminus \mathbb{N}$ a nonstandard element of the universe of $\mathfrak{N}$ which is definable.

4. Let $\mathfrak{A}$ be an $\mathcal{L}$-structure and two elements a and b of the universe of $\mathfrak{A}$. Show that the following are equivalent.

(a) There is a definable function f for which $f(a) = b$.

(b) For any elementary extension $\mathfrak{B} \succeq \mathfrak{A}$ and automorphism $\sigma : \mathfrak{B} \to \mathfrak{B}$, if $\sigma(a) = a$, then $\sigma(b) = b$.

5. Let $\mathcal{L} = \mathcal{L}(U, V)$ be the first-order language having exactly two unary predicate symbols, U and V, and no other nonlogical symbols. Describe all the of the complete theories in $\mathcal{L}$. You should show that the theories you propose are distinct and that they exhaust all of the completions.

6. Let $\{W_e\}_{e \in \omega}$ be the usual enumeration of the recursively enumerable sets. Show that $\mathsf{Fin} := \{e \in \omega : W_e \text{ is finite } \}$ is Σ^0_2-complete.

7. let $\mathfrak{A} = (U, I, f, g, \ldots)$ be a structure for a finite language $\mathcal{L}$, where I is a unary relation and f and g are binary functions. Let π be an isomorphism from $(\mathbb{N}, +, \times)$ to $(I, f \restriction I, g \restriction I)$. For ϕ an $\mathcal{L}$-sentence, let $\mathsf{GN}(\phi)$ be the Gödel number of ϕ in some reasonable Gödel numbering.

Show that $\{\pi(\mathsf{GN}(\phi)) | \phi \text{ is an } \mathcal{L}\text{-sentence and } \mathfrak{A} \models \phi\}$ is not definable over $\mathfrak{A}$ without parameters.

8. Give an example of a pair of first order languages $\mathcal{L} \subseteq \mathcal{L}'$ and complete theories $T \subseteq T'$ in $\mathcal{L}$ and $\mathcal{L}'$, respectively, for which T' is $\aleph_1$-categorical but T has more than one model of cardinality $\aleph_1$. [Prove that your proposed example works.]

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